Numerical Continuation and Bifurcation Tracking Using the Harmonic Balance Method

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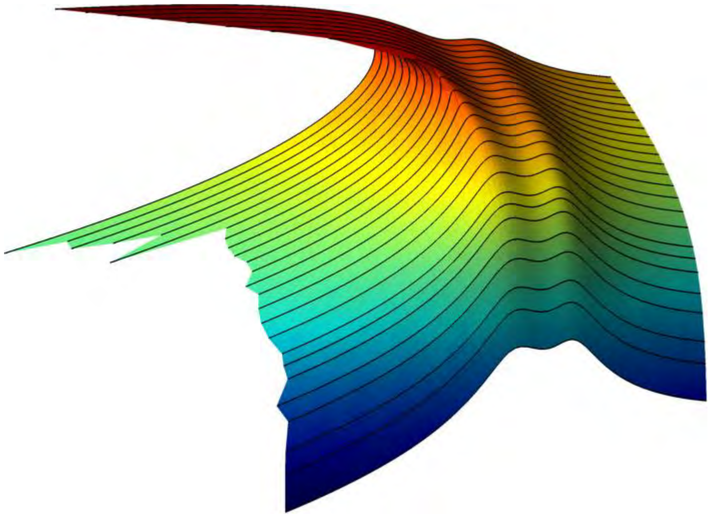
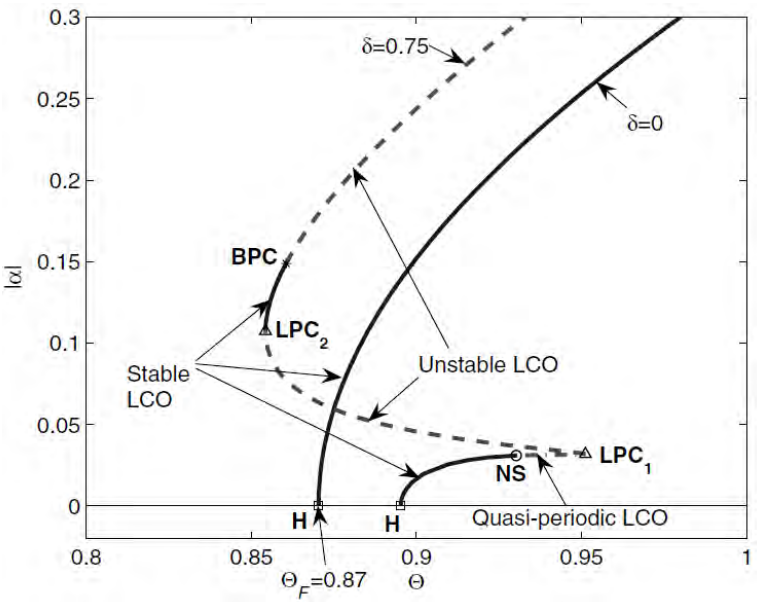


Bifurcations: Key Information About Nonlinear Systems

They can trigger instabilities:



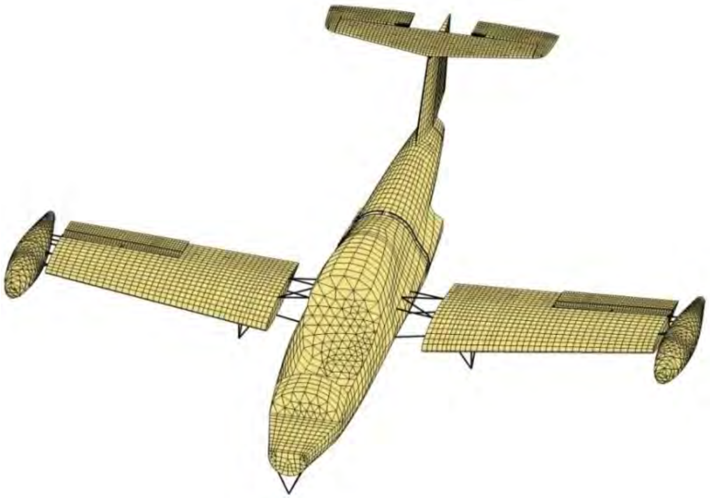
They also occur near resonance peaks

- How to eliminate these bifurcations, to master their effects?

How to Track Bifurcations of Large Structures?

Solution #1: Orthogonal collocation

Memory issues…



Solution #2: Shooting technique

Numerous time integrations…

Solution #3: Harmonic Balance (HB) method

Interesting filtering property!

Outline

Harmonic balance method and bifurcation tracking

Application to a nonlinear tuned vibration absorber

Application to a large-scale structure

Challenges & Conclusions

Harmonic Balance Method…

$$M\ddot{X} + C\dot{X} + KX = F_L(\omega,t) - F_{NL}(X,\omega,t) = F(X,\omega,t)$$

Solution *X(t)* and forces *F(X,ω,t)* decomposed in Fourier series

Z

$$X(t) = B_0 + \sum_{k=1}^{N_H} \left(A_k \sin(k\omega t) + B_k \cos(k\omega t) \right)$$

New unknowns: Fourier coefficients Z

b

$$F(Z,\omega,t) = C_0 + \sum_{k=1}^{N_H} \left(S_k \sin(k\omega t) + C_k \cos(k\omega t) \right)$$

Coefficients b depend on Z

…An Appropriate Tool for Large Structures

Equations of amplitude

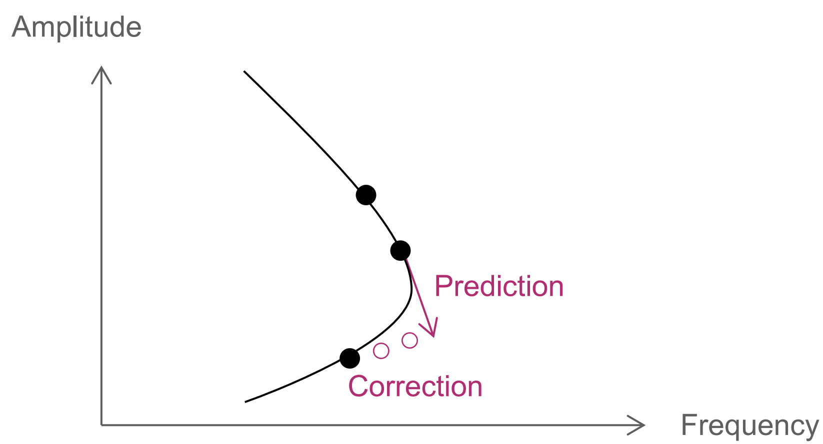
$$H(Z) \equiv A(\omega)Z - b(Z,\omega) = 0$$

with

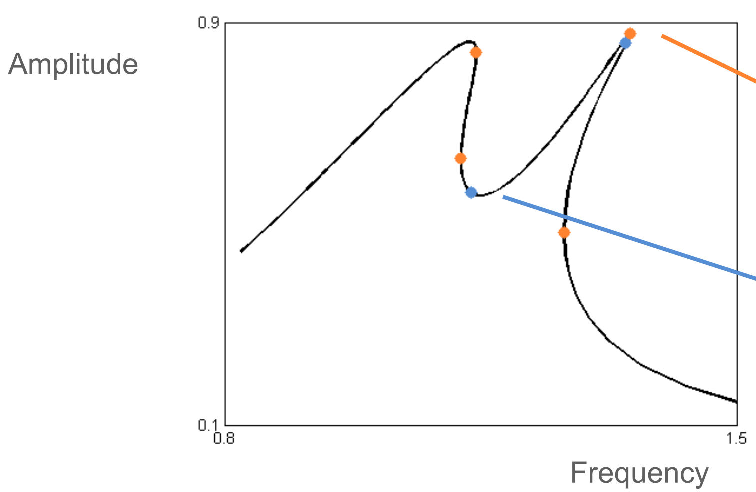
$$A = \begin{bmatrix} K & & & & & & \\ & K-\omega^2 M & -\omega C & & & & \\ & \omega C & K-\omega^2 M & & & & \\ & & & \ddots & & & \\ & & & & K-(k\omega)^2 M & -(k\omega)C & \\ & & & & (k\omega)C & K-(k\omega)^2 M & \\ & & & & & & \ddots \end{bmatrix}$$

How to Track and Analyze Periodic Solutions?

Continuation method



Detection of bifurcations on the frequency response



Limit point (LP) bifurcations

Jump phenomenon

Naimark-Sacker (NS) bifurcations

Coexisting quasiperiodic solutions

How to Track LP Bifurcations with HB Method?

Extra equation

$$\det(J) = 0 \quad \text{where} \quad J = \frac{\partial H}{\partial Z}$$

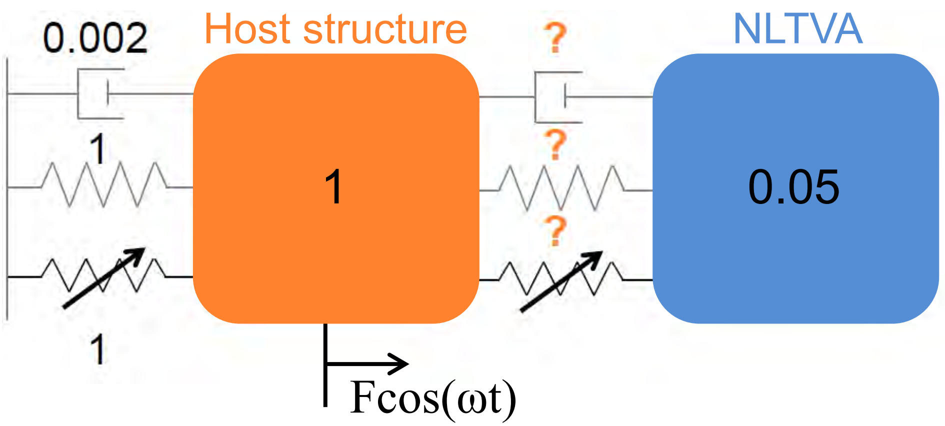
Bordering technique to improve numerical performance

$$\begin{bmatrix} J & p \\ q^* & 0 \end{bmatrix} \begin{bmatrix} w \\ g \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{so that} \quad g = 0 \Leftrightarrow \det(J) = 0$$

Full system

$$\begin{cases} H = 0 \\ g = 0 \end{cases}$$

Application #1: The Nonlinear Tuned Vibration Absorber



Linear tuning: Den Hartog's Equal Peaks rule (LTVA)

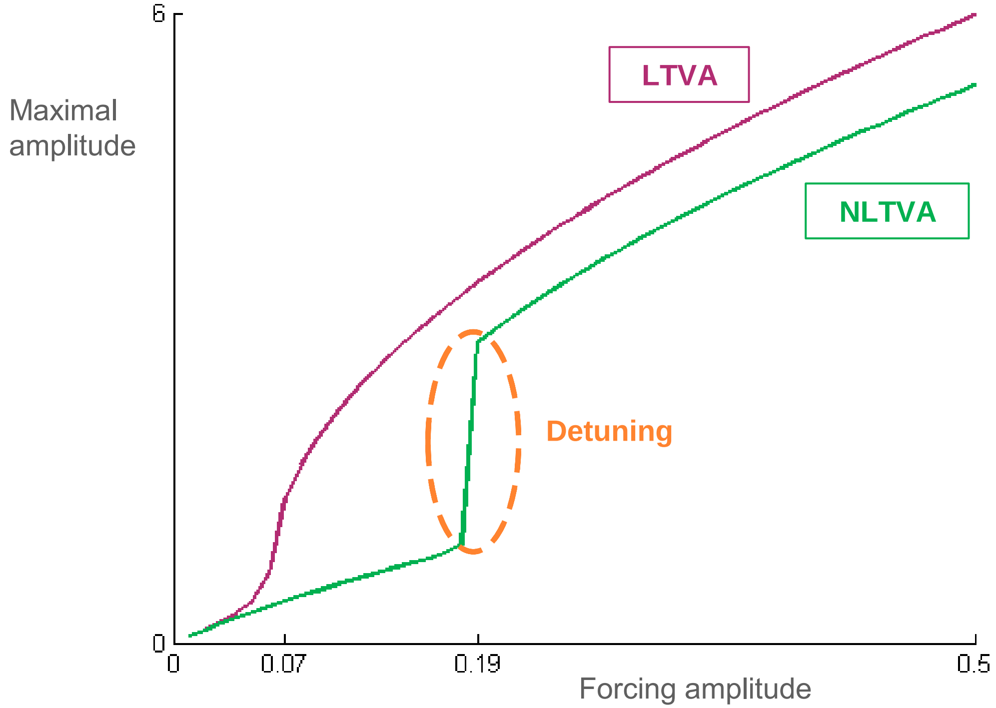
$$k_{abs} = 0.0453, \quad c_{abs} = 0.013$$

Nonlinear tuning based on: - Frequency-Energy dependence of the host structure

- Generalized Equal Peaks rule for different forcing levels

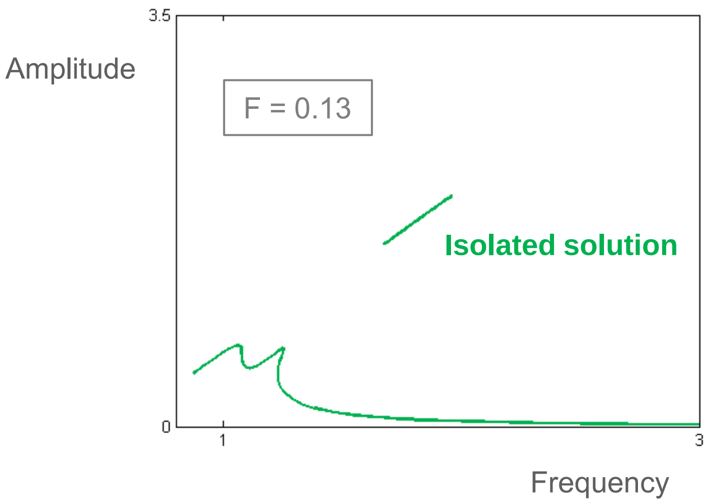
$$f_{nl} = 0.0042 x^3$$

The NLTVA Always Performs Better than the LTVA



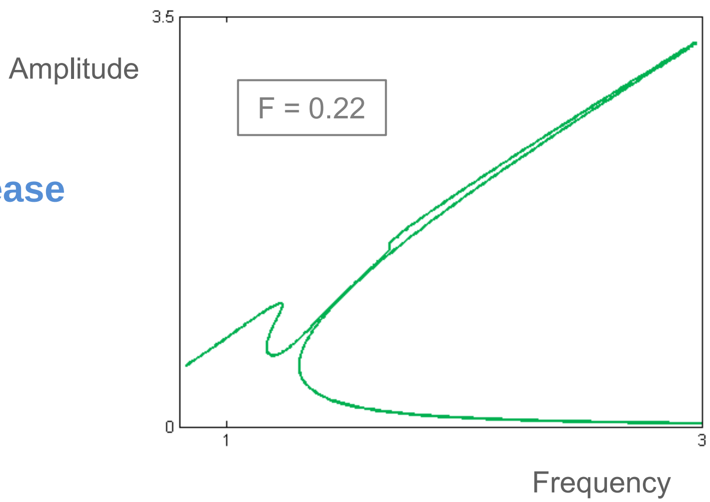
How Does the NLTVA Get Detuned?

Merging of the main frequency response and an isolated solution

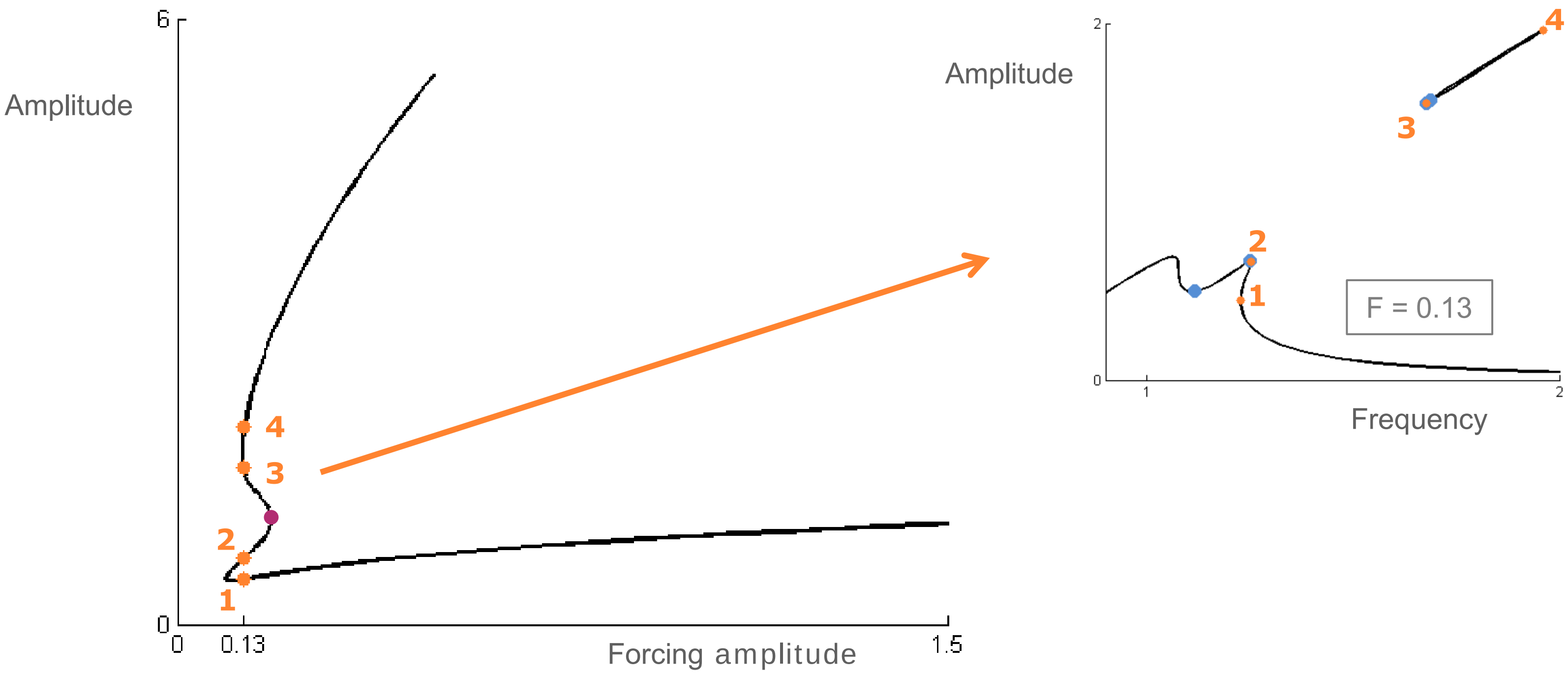


Forcing increase

(Detuning)



Tracking of LPs as a Tool for Global Analysis

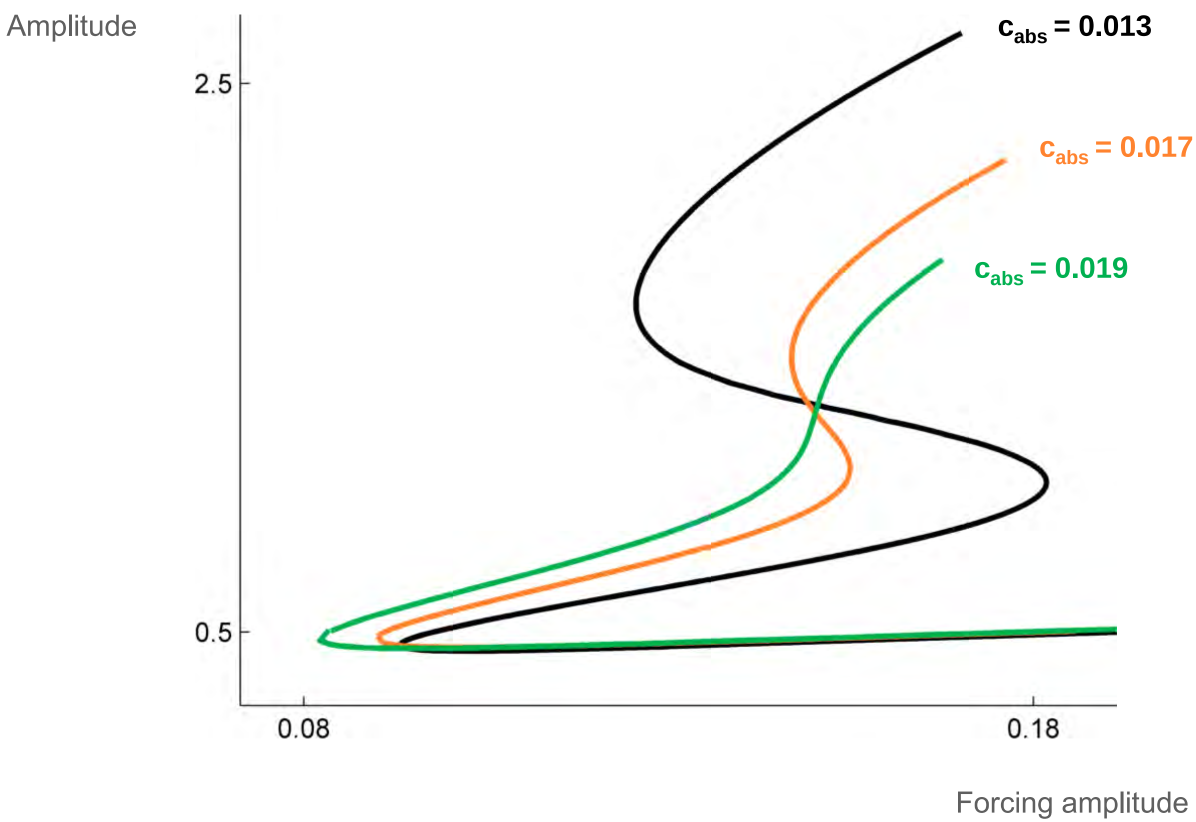


The detuning is non smooth because of the limit point

Is it possible to “smooth” it by changing c_{abs}?

Effect of c_{abs} on the LP branches

Smooth detuning for c_{abs} > 0.019



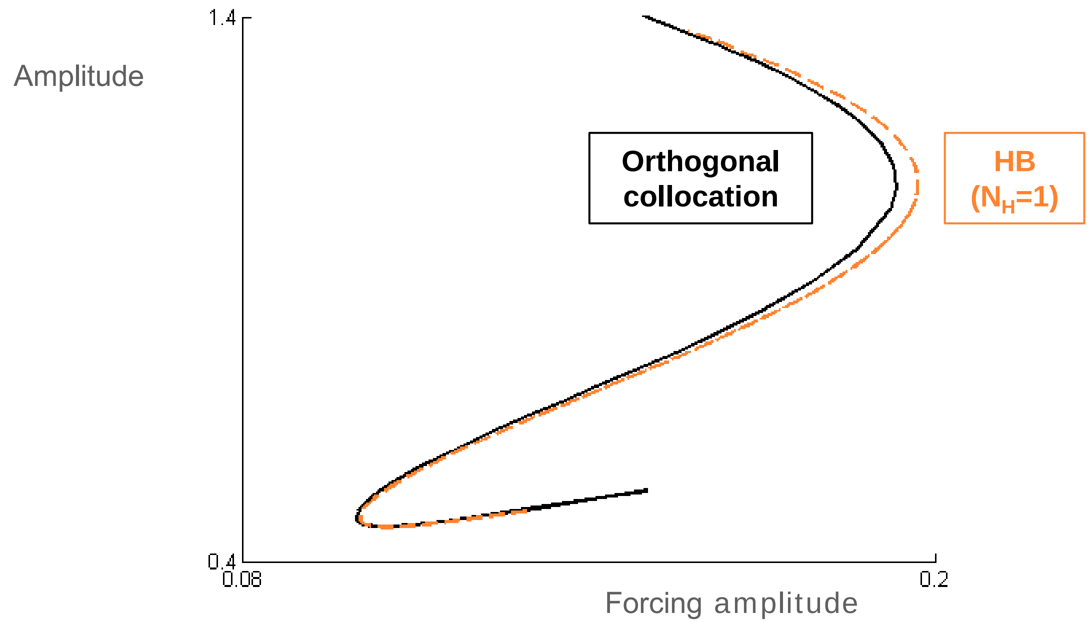
Challenges: Tracking of Naimark-Sacker Bifurcations

NS bifurcation for the equations of motion

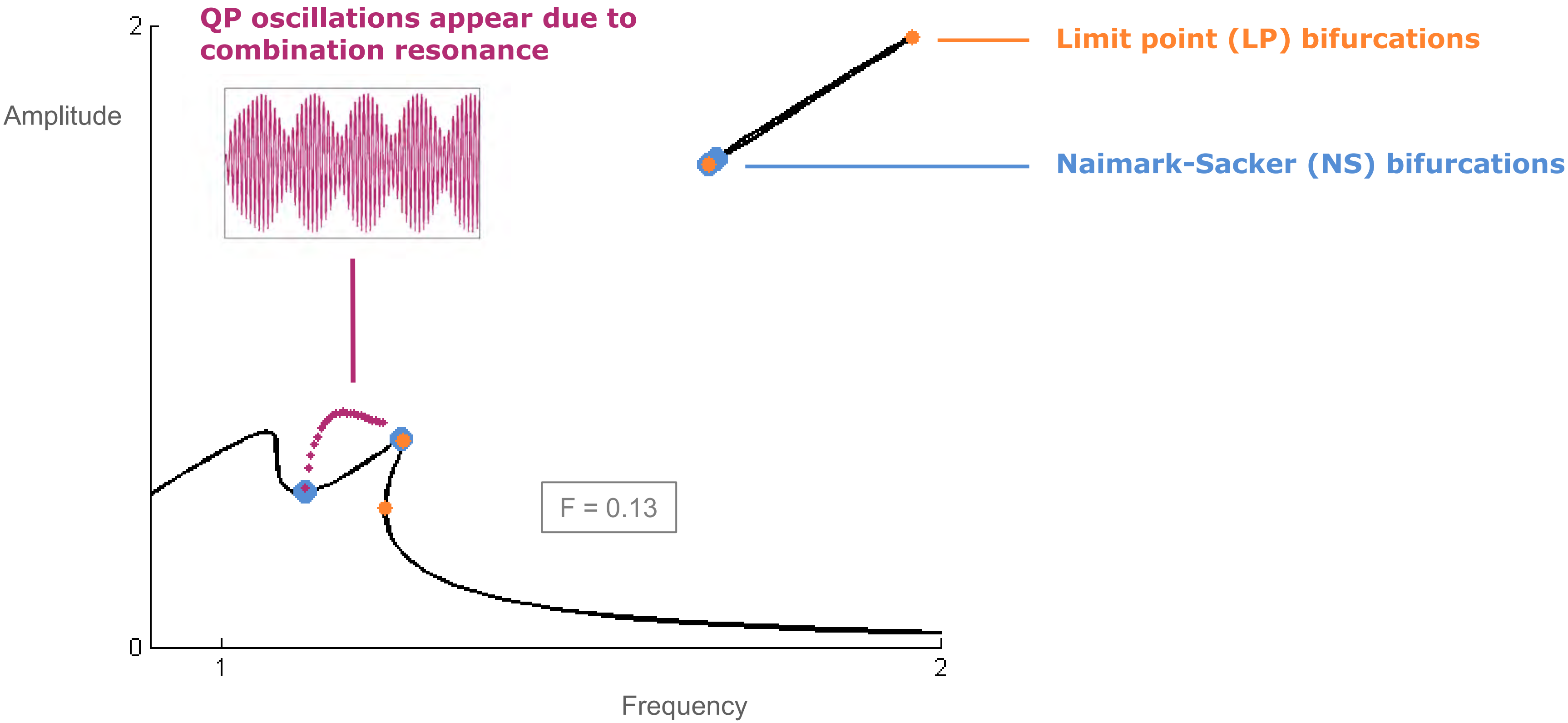
$$M\ddot{X} + C\dot{X} + KX = F_L(\omega,t) - F_{NL}(X,\omega,t) = F(X,\omega,t)$$

≈ Hopf bifurcation for the equations of amplitude

$$H(Z) \equiv A(\omega)Z - b(Z,\omega) = 0$$

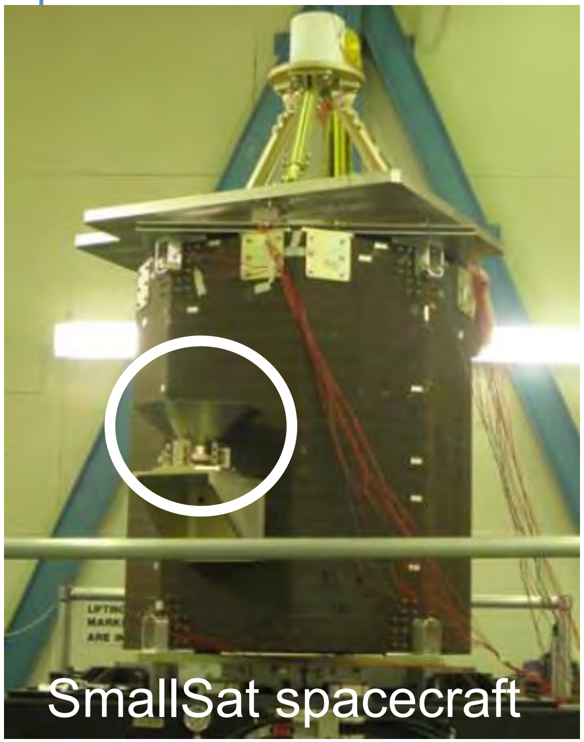


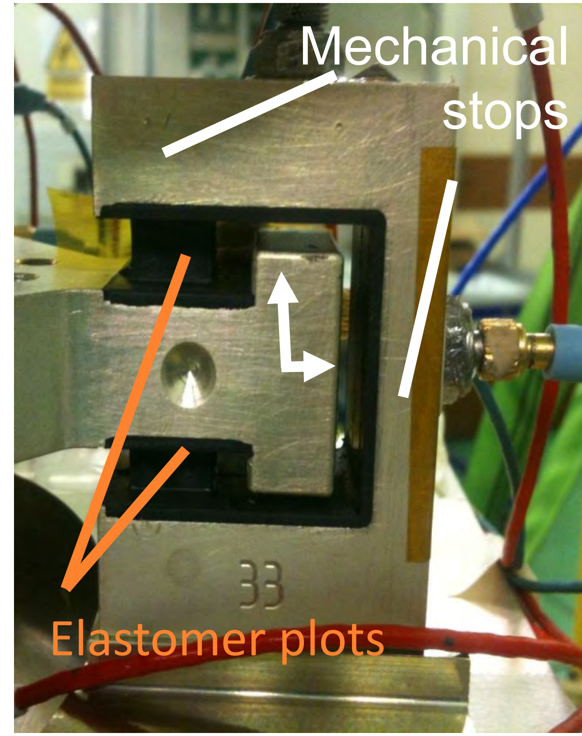
Challenges: Quasiperiodic (QP) Oscillations



How to carry out the continuation of **QP oscillations** with the **harmonic balance method**?

Challenges: Large-Scale Structures – SmallSat

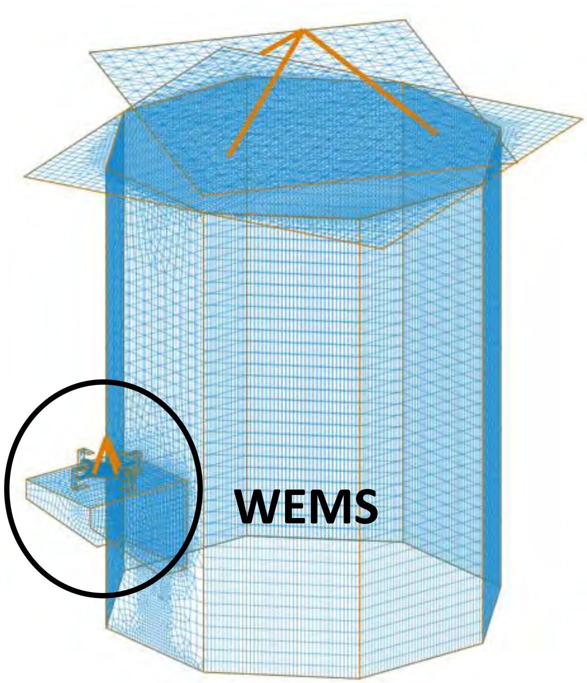




Goals	Solutions
Micro-vibration mitigation	Elastomer plots
Large amplitude limitation	Mechanical stops

Challenges: Large-Scale Structures – SmallSat

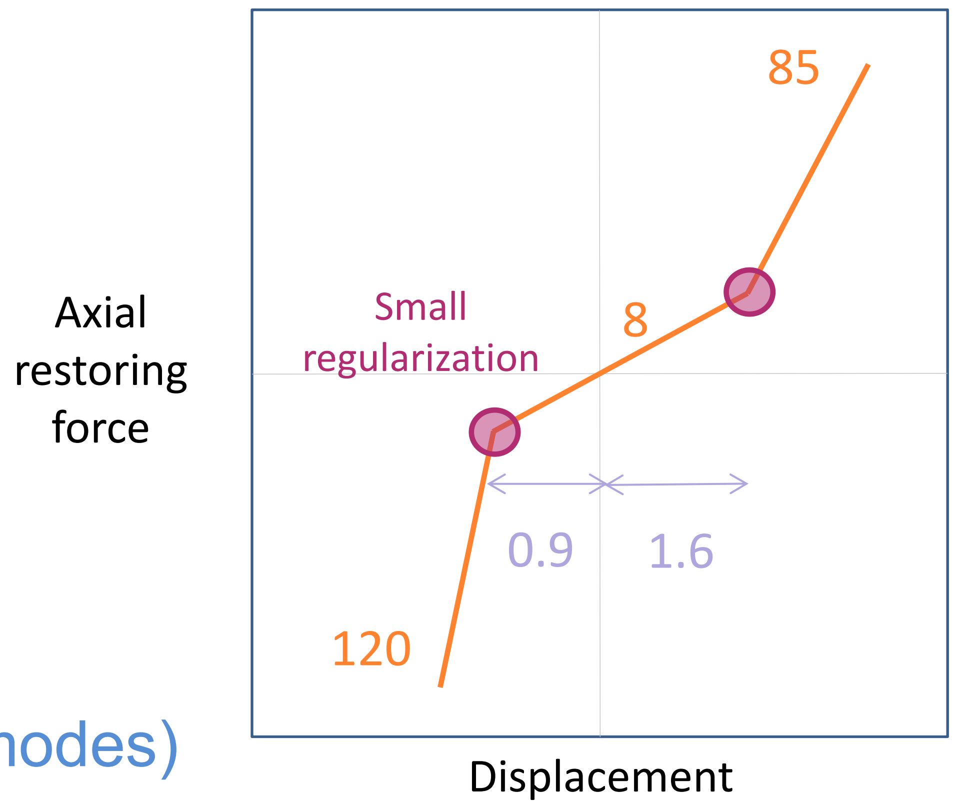
Detailed FE model



Reduced FE model:

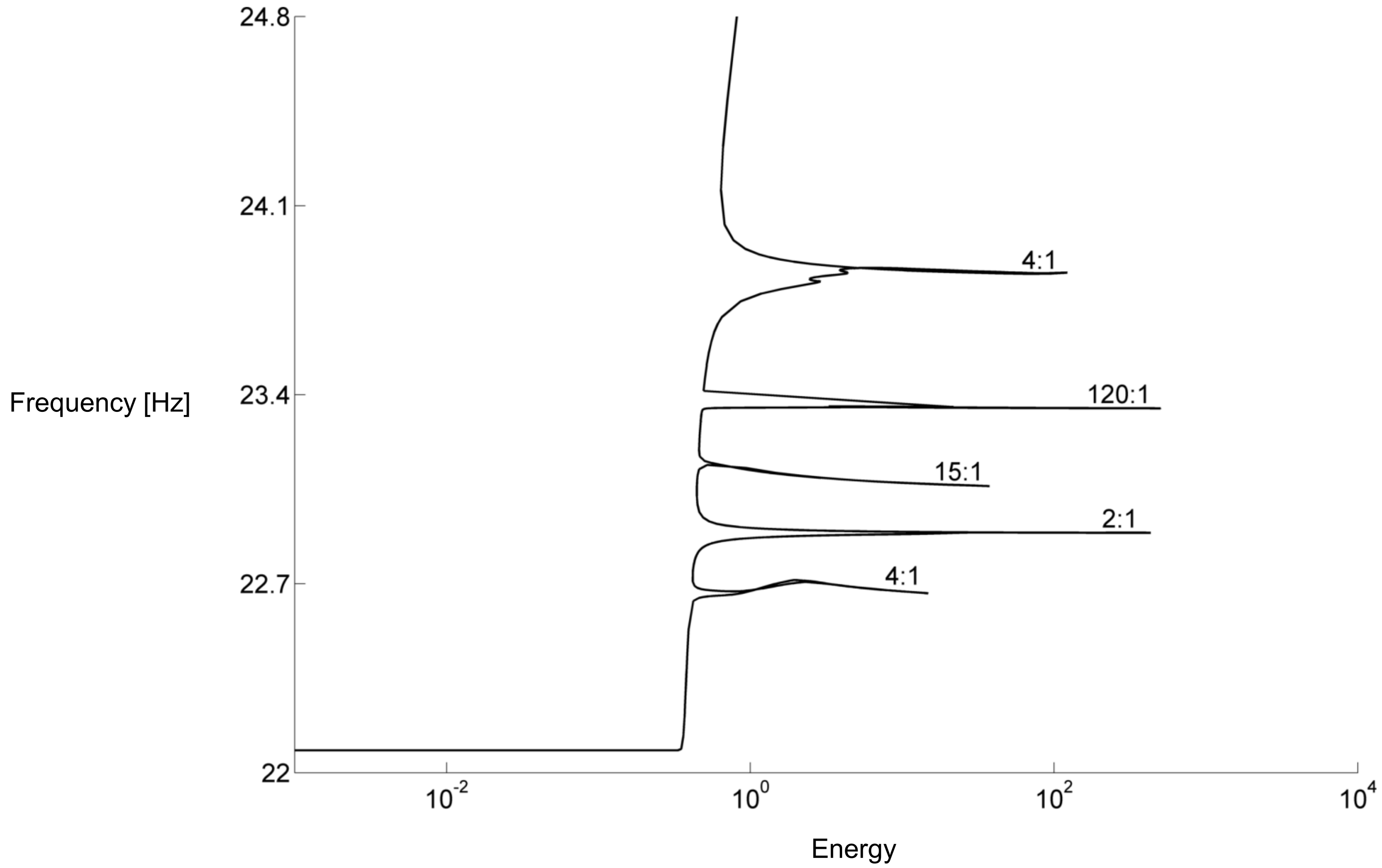
34 DOFs (24+10 internal modes)

8 piecewise nonlinearities identified experimentally (asymmetry due to gravity)



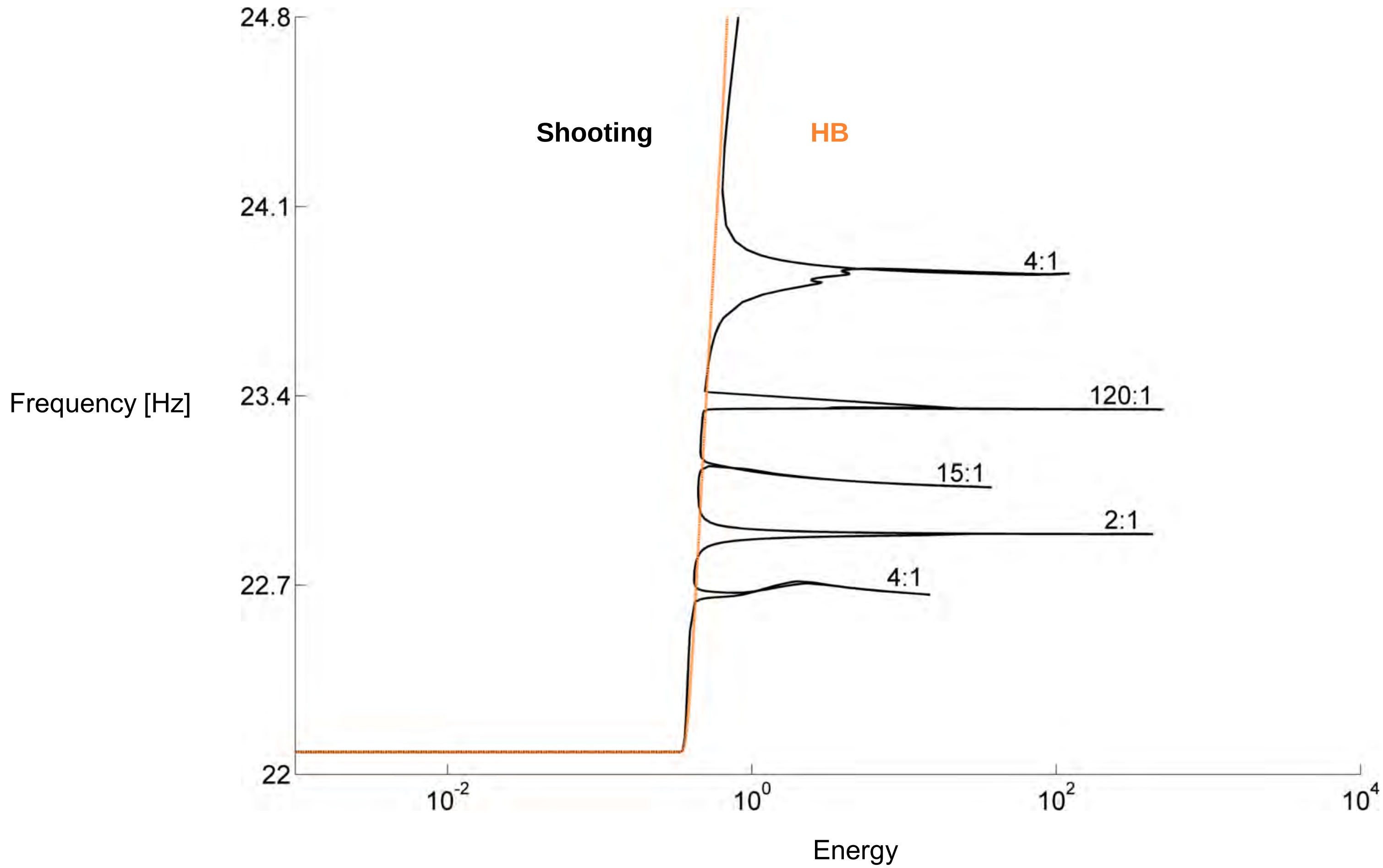
Nonlinear Normal Modes (NNMs) of the SmallSat

Shooting



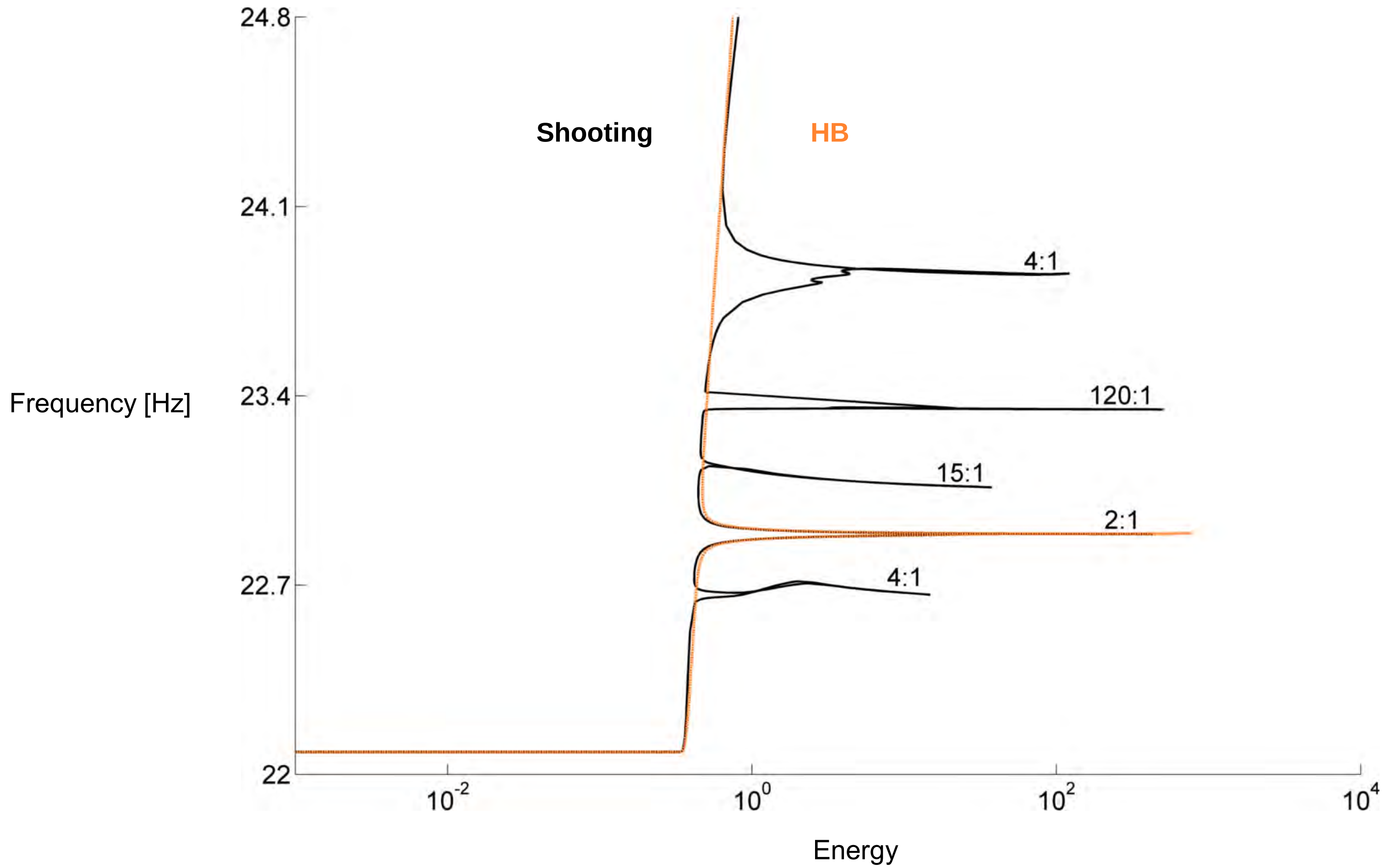
Nonlinear Normal Modes (NNMs) of the SmallSat

$N_H = 1$ – Filtering of the internal resonances



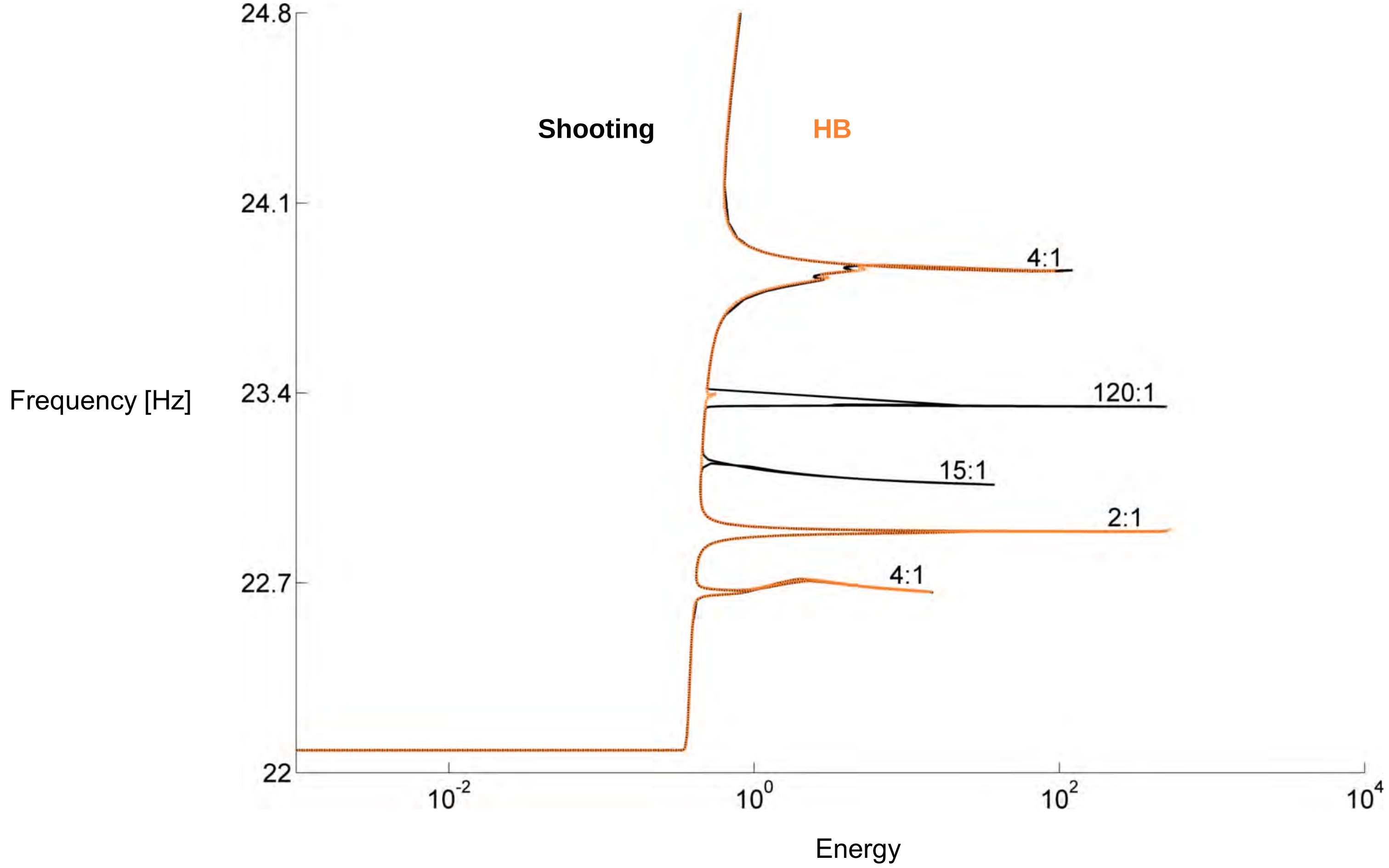
Nonlinear Normal Modes (NNMs) of the SmallSat

$N_H = 3$ – Filtering of the internal resonances



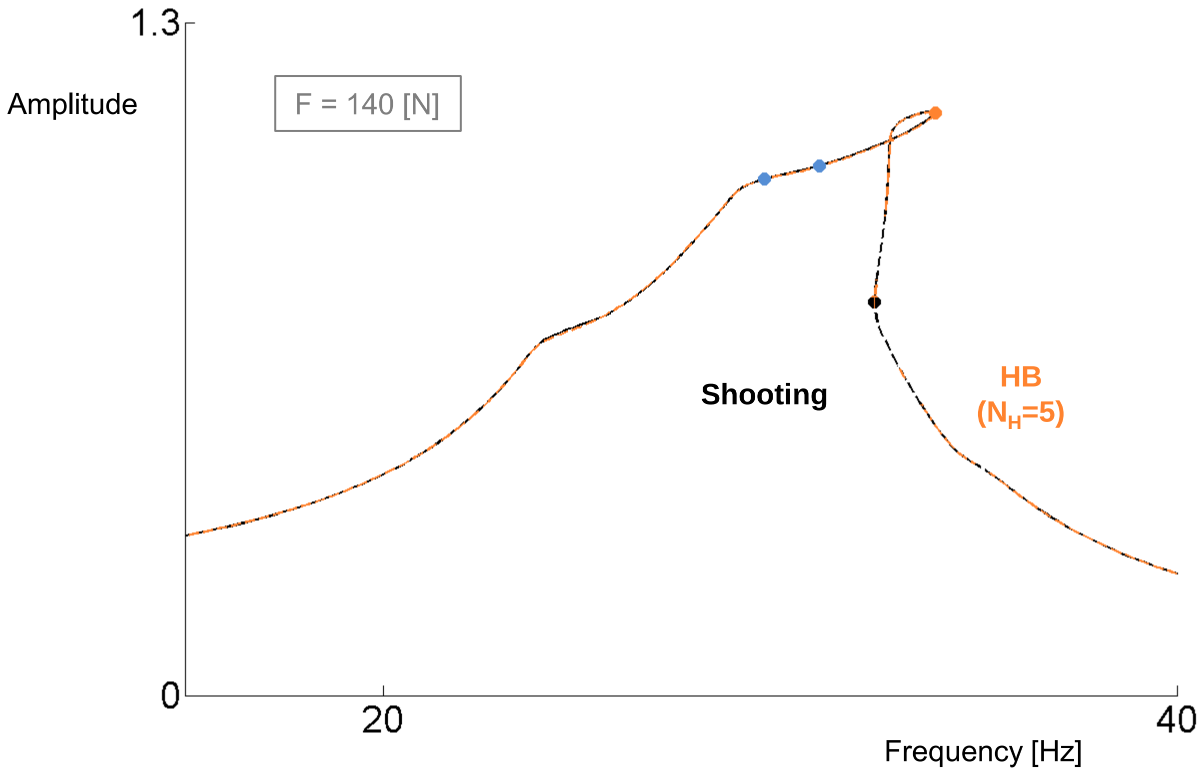
Nonlinear Normal Modes (NNMs) of the SmallSat

$N_H = 5$ – Filtering of the internal resonances



Response of the SmallSat to Harmonic Forcing

Good approximation with only 5 harmonics



Conclusions: Harmonic Balance Method

Numerical continuation of periodic solutions +

Bifurcation tracking

Promising tool for analyzing and designing nonlinear systems

Suitable for large structure and for strong nonlinearities

Interesting filtering properties

Thank you for your attention.

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