Transverse subharmonic vibration

of an axially-excited beam

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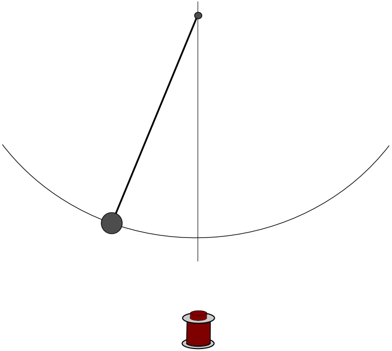
October 14-15, 2015
DYNOLIN Conference, Lyon (France)

Definition of the argumentary oscillator.



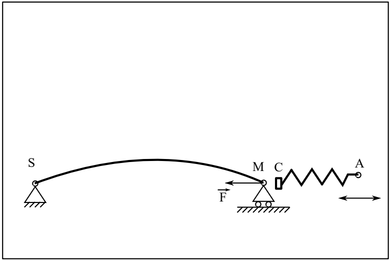
- History: Béthenod, Penner, Doubochinski.
- Oscillator often of Duffing type.
- Submitted to an external periodic force.
- Modulation-coupling by spatial position.
- Equation (reduced time):

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + x + \mu x^3 = A\ H(x)\ F(t)$$

with F(t)=periodic function of time.

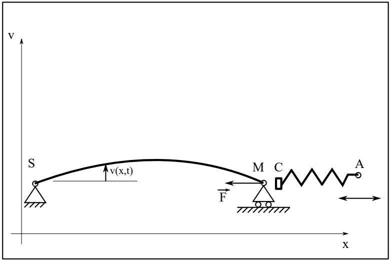
- Modelling to find the H-function.

Setup.



- Beam:
  - Articulated on the left (point S)
  - Articulated-guided on the right (point M)
- Point A:
  - Forced harmonic horizontal movement
  - Linked to C by a linear spring
  - C in partial contact with M

PDE.



Equation of the beam's motion:

$$\rho S\ddot{v} + EIv^{(4)} - Fv^{(2)} + 2\beta\dot{v} = f$$

where

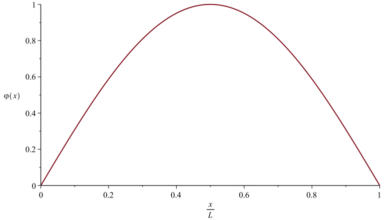
- x = longitudinal coordinate
- $v(x, t)$ = transverse coordinate
- ρ = density,
- S = section area,
- e = Yung's modulus,
- I = quadratic moment,
- $F(t)$ = axial force applied to M,
- β = damping,
- $f(x, t)$ = transverse force's linear density.

F force.

$F(x_M, t) = F_0 + k(l(t) - l_0)$
where

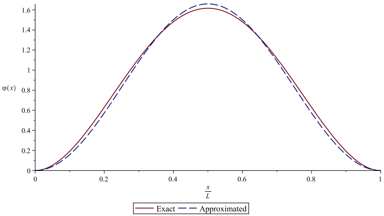
- $l(t) =$ actual spring length
- $l_0 =$ idle spring length
- $k =$ spring stiffness
- $F_0 =$ force applied to the beam when beam is in rectilinear position and point A at center position

First-mode shapes.



Case articulated-(articulated-guided):

$$q_1(x) = \sin\left(\frac{\pi x}{L}\right)$$

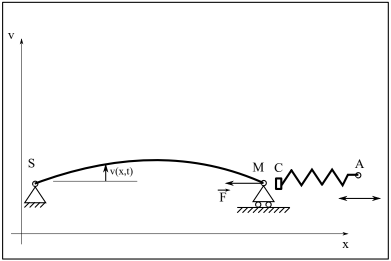


Case clamped-(clamped-guided):

$$q_1(x) \approx b\left(1 - \cos\left(\frac{2\pi x}{L}\right)\right)$$

with $b = 0.83$

M's abscissa.



Inextensible beam $\rightarrow s(x_M, v, t) = L$:

$$s(x_M, v, t) = \int_0^{x_M} \sqrt{1 + \left(\frac{\partial v(u,t)}{\partial u}\right)^2} du = L$$

$$\Rightarrow x_M(t) \approx L - \frac{1}{2}\int_0^L \left(\frac{\partial v(u,t)}{\partial u}\right)^2 du$$

Case articulated-(articulated-guided), first mode :

$$v(x,t) = Lq_1(t)\sin\left(\pi\frac{x}{L}\right) \Rightarrow x_M(t) \approx L\left(1 - \frac{\pi^2}{4}q_1^2(t)\right)$$

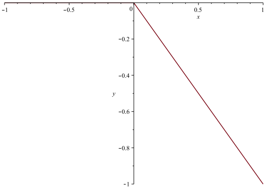
Case clamped-(clamped-guided), first mode :

$$v(x,t) \approx b\left(1 - \cos\left(\frac{2\pi x}{L}\right)\right) \Rightarrow x_M(t) \approx \; L\left(1 - \pi^2 q_1^2(t)\right)$$

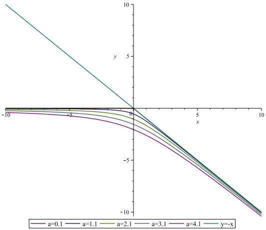
Contact law.

Principle: soft transition between contact and non-contact, by smoothing the curve of $y(x) = -\dfrac{x+|x|}{2}$.

Purpose: avoid solver's discrepancies.



$$y = -\frac{x+|x|}{2} \quad \text{a} \quad C^0 \text{ function.}$$



$$y = -\frac{x+\sqrt{x^2+a^2}}{2}, \quad \text{a} \quad C^\infty \text{ function.}$$

$a = 0.1 \ldots 4.1$ in figure.

Modal projection.

For the p-th modal coordinate, case articulated-(articulated-guided):

$$\ddot{q}_p(t) + 2\frac{\beta}{\rho S}\dot{q}_p(t) + \left(\frac{p\pi}{L}\right)^4 \frac{EI}{\rho S} q_p(t) + \left(\frac{p\pi}{L}\right)^2 \frac{EI}{\rho S} F(q_1, t) q_p(t) - \frac{2}{L^2 \rho S}\int_0^L \sin\left(\frac{p\pi}{L}x\right) f(x,t)dx = 0$$

For the first mode, case articulated-(articulated-guided):

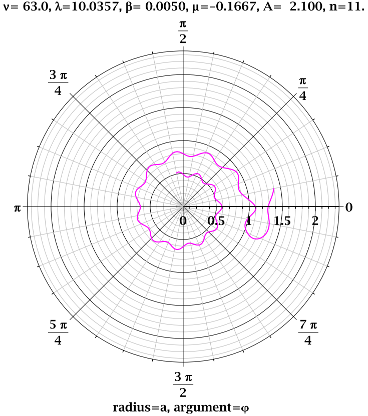
$$\ddot{q}_1(t) + 2\frac{\beta}{\rho S}\dot{q}_1(t) + \left(\frac{\pi}{L}\right)^4 \frac{EI}{\rho S} q_1(t) + \left(\frac{\pi}{L}\right)^2 \frac{EI}{\rho S} F(q_1, t) q_1(t) - \frac{2}{L^2 \rho S}\int_0^L \sin\left(\frac{\pi}{L}x\right) f(x,t)dx = 0$$

For the first mode, case clamped-(clamped-guided):

$$\ddot{q}_1(t) + 2\frac{\beta}{\rho S}\dot{q}_1(t) + \left(2\frac{\pi}{L}\right)^4 \frac{EI}{3\rho S} q_1(t) + \left(2\frac{\pi}{L}\right)^2 \frac{EI}{3\rho S} F(q_1, t) q_1(t) - \frac{2}{3L^2 \rho S}\int_0^L \left(1 - \cos\frac{2\pi}{L}x\right) f(x,t)dx = 0$$

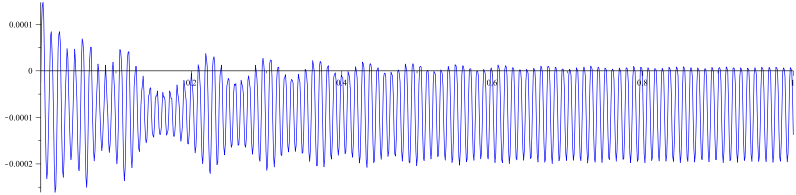
In the simulations, we assume $f(x,t) \equiv 0$.

Van der Pol amplitude/phase representation.

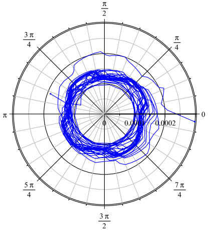


- The averaging method searches an approximate solution $x = a(t)\ \sin(\omega t + \varphi(t))$, where $a(t)$ and $\varphi(t)$ are slowly-varying functions of time.
- (a, φ) –> Natural representation: van der Pol.
- Example of integral curve winding up around the origin.
- For the simulations: $n = \dfrac{\textit{Excitation frequency}}{\textit{Oscillator frequency}} =$ 4 to 10 (even integer), $\beta = 0.0001..0.01$.
- Polar form is better, due to invariance vs $\dfrac{2\pi}{n}$ rotation. Radius=a, argument=φ.

Preliminary simulation results: result #1.



Equation 2nd ordre d'origine, extraction via x et x_point, ae0=0.0003, φ0=-0.0873009182, decalage_phi0=-0.8, A=0.0004, ph. offset=0., n=4, nu excitation=1990.041208, omega final oscillateur estimée=497.5103019, periode1=0.01262925669, β=0.01, μ=10000.00000, t_max=1, nb pnts=1000

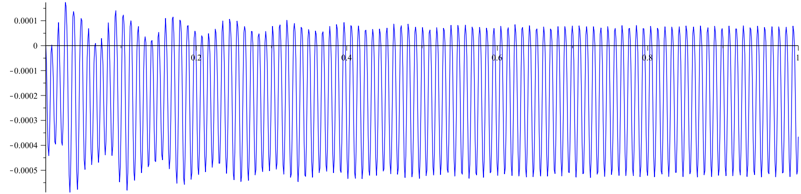


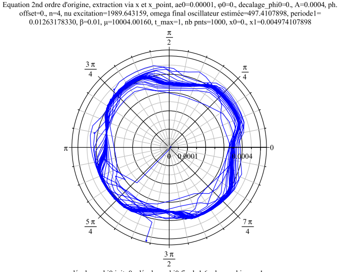
Simulation duration: 1 s
Buckling force: 6900 N
Force in point A: 3 N max

$f_0 = 80\ Hz,\ \dfrac{\nu}{\omega} = 4,\ \dfrac{\omega}{\omega_0} = 1.01$

$E = 70\ 10^9$ Pa, $\rho = 2700\ kg/m^3$, $I = 10^{-8}\ m^4$,
$S = 10^{-4}\ m^2$, $L = 1\ m$, $\beta = 10^{-2}$,
Point A's amplitude $= 4\ 10^{-7}\ m$,
$F_0 = 0.0\ N$.

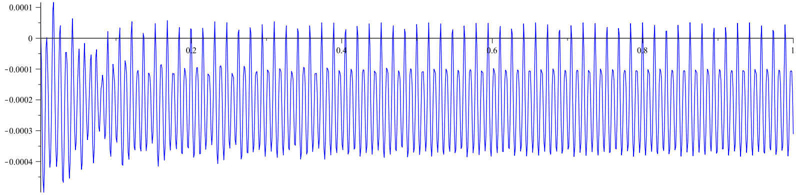
Preliminary simulation results: result #2.

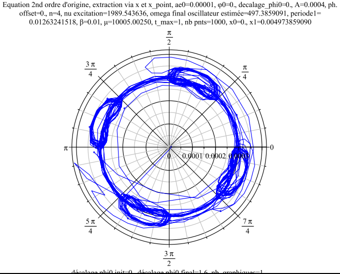




Simulation duration: 1 s
Buckling force: 6900 N
Force in point A: 5.6 N max
$f_0 = 80\ Hz,\ \dfrac{\nu}{\omega} = 4,\ \dfrac{\omega}{\omega_0} = 1.01$
$E = 70\ 10^9$ Pa, $\rho = 2700\ kg/m^3$, $I = 10^{-8}\ m^4$,
$S = 10^{-4}\ m^2$, $L = 1\ m$, $\beta = 10^{-2}$,
Point A's amplitude $= 4\ 10^{-7}\ m$,
$F_0 = -2.7\ N$.

Preliminary simulation results: result #3.



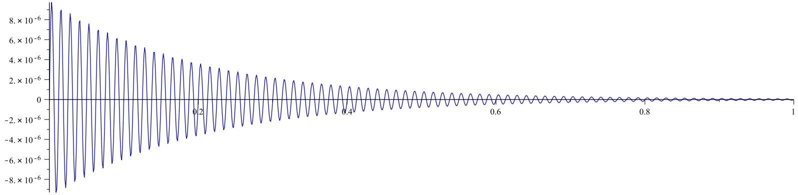


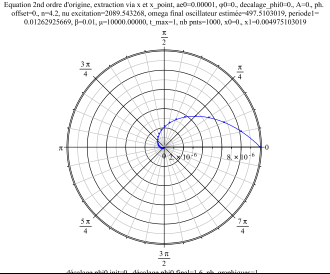
Simulation duration: 1 s
Buckling force: 6900 N
Force in point A: 6.2 N max

$f_0 = 80\ Hz,\ \dfrac{\nu}{\omega} = 4,\ \dfrac{\omega}{\omega_0} = 1.01$

$E = 70\ 10^9$ Pa, $\rho = 2700\ kg/m^3$, $I = 10^{-8}\ m^4$,
$S = 10^{-4}\ m^2$, $L = 1\ m$, $\beta = 10^{-2}$,
Point A's amplitude $= 4\ 10^{-7}\ m$,
$F_0 = -3.5\ N$.

Preliminary simulation results: result #4.





Simulation duration: 1 s
Buckling force: 6900 N
Force in point A: 0 N max

$f_0 = 80\ Hz,\ \frac{\nu}{\omega} = 4,\ \frac{\omega}{\omega_0} = 1.01$

$E = 70\ 10^9$ Pa, $\rho = 2700\ kg/m^3$, $I = 10^{-8}\ m^4$,
$S = 10^{-4}\ m^2$, $L = 1\ m$, $\beta = 10^{-2}$,
Point A's amplitude $= 0\ m$,
$F_0 = 0\ N$.

Future work

- Simulations based on complete original motion PDE.
- Other contact conditions.
- Experimentation.

Thank you for your attention.