Calcul des modes non linéaires avec Manlab-4.0

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Ecole Thématique - Dynolin 2018

Outline

1 Overview

2 Continuation of periodic solutions with Manlab-4.0

Summary of the method :

- Find the periodic solution of an ODE system

$$\dot{Y} = f(Y, \lambda) \qquad Y(t) \in \mathbb{R}^n$$

by using a high order Fourrier serie expansion (HBM)

$$Y(t) = Y_0 + \sum_{k=1}^{H} Y_{c,k} \cos(k\omega t) + \sum_{k=1}^{H} Y_{s,k} \sin(k\omega t)$$

- The resulting (nonlinear) algebraic system on the Fourrier coefficients reads :

$$R(U) = 0 \qquad \text{with} \qquad U = [Y_0, Y_{c,k}, Y_{s,k}, \omega, \lambda]$$

- Path following of the branches using high order Taylor series expansions with respest to a path parameter a .

$$U(a) = U_0 + a\, U_1 + a^2 \; + \cdots + a^N\, U_N$$

- The resulting linear algebraic system on the U_i reads :

$$\frac{\partial \mathbf{R}}{\partial \mathbf{U}} \mathbf{U}_p = F_p^{nl}(\mathbf{U}_1, \ldots, \mathbf{U}_{p-1})$$

The conerstone

- In HBM : insert $\theta(t) = \theta_0 + \sum_{k=1}^{H} \theta_{c,k} \cos(k\omega t) + \sum_{k=1}^{H} \theta_{s,k} \sin(k\omega t)$ into, for example,

$$R(\theta(t), \lambda) := \ddot{\theta} + \lambda\dot{\theta} + \theta^3 + \sin(\theta) = 0$$

  and "balance" the harmonic !

- in ANM : insert $u(a) = u_0 + a\, u_1 + a^2 \ + \cdots + a^N\, u_N$ into

$$R(u, \lambda) := u + u^2 + \frac{tan(u)}{1+u} - \lambda = 0$$

  and collect term with the same powers !

How to :

- use Automatic Differentiation to do the job : nice for the user but poor efficiency.
- do a quadratic recast of the equation : then the job become easy and efficient

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Back to Fourier series : quadratic recast for HBM

Framework for ODE :

- To find the periodic solution of the ODE system

$$\dot{Y} = f(Y, \lambda) \qquad\qquad Y \in \mathbb{R}^n$$

- first, recast quadratic $Z = [Y; Y_{aux}]$ $\rightarrow$ quadratic DAE system

$$m(\dot{Z}) = c(\lambda) + l(Z, \lambda) + q(Z, Z)$$

- then apply Fourrier serie expansion (HBM)

$$Z(t) = Z_0 + \sum_{k=1}^{H} Z_{c,k} \cos(k\omega t) + \sum_{k=1}^{H} Z_{s,k} \sin(k\omega t)$$

- The resulting algebraic system on the Fourrier coefficients is of the form

$$R(\hat{Z}, \omega, \lambda) = C(\lambda) + L(\hat{Z}, \lambda) + Q(\hat{Z}, \hat{Z}) - \omega M(\hat{Z}) \qquad \text{with} \qquad \hat{Z} = [Z_0, Z_{c,k}, Z_{s,k}]$$

Back to Fourier series : quadratic recast for HBM

If

- $C(lambda)$ is quadratic in λ
- $L(Z,\lambda)$ is linear in λ

Then

$$R(\hat{Z},\omega,\lambda) = C(\lambda) + L(\hat{Z},\lambda) + Q(\hat{Z},\hat{Z}) - \omega M(\hat{Z}) \qquad \text{is quadratic !}$$

Advantage of this (ODE)framework : clear, stability available.

Drawback : double size system for mechanical systems with second time derivative

Back to Fourier series : quadratic recast for HBM

Framework for Nonlinear Mode of mechanical conservative system :

- equation of a mechanical conservative system

$$\mathbf{M}\ddot{\mathbf{U}} + f_{nl}(\mathbf{U}) = 0$$

- add artificial damping to fall down into the continuation framework

$$\mathbf{M}\ddot{\mathbf{U}} + \lambda\dot{\mathbf{U}} + f_{nl}(\mathbf{U}) = 0$$

Manlab-4.0 can take into account of these two terms $\mathbf{M}\ddot{\mathbf{U}}$ and $\lambda\dot{\mathbf{U}}$

However, stability analysis no more available

TP Matlab for periodic solutions

Good Luck ! ! !