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# Nonlinear Modal Analysis of Conservative and Nonconservative Aerospace Structures

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by

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# Abstract

The concept of nonlinear normal modes (NNMs) provides a solid and rigorous theoretical framework for the analysis of the nonlinear oscillations of mechanical systems. If NNMs have been studied since more than fifty years, it is only very recently that contributions dealing with their numerical calculation have been reported in the literature. Although these methods pave the way for the application of NNMs to more complex systems, they have not yet reached the necessary maturity. In this context, the purpose of this research is (i) to further investigate the performance of an existing method for computing the NNMs of conservative systems and (ii) to propose two new methods for the computation of NNMs of nonconservative systems.

The first contribution of this thesis is to calculate the NNMs of a real-life aerospace structure, the SmallSat spacecraft developed by EADS Astrium. An algorithm that combines an advanced shooting method with the pseudo-arclength continuation technique is utilized. We show that the NNMs provide a very useful interpretation of the strongly nonlinear dynamics of the spacecraft. One specific contribution is to numerically reproduce with great fidelity several interactions between modes with noncommensurate linear frequencies that were observed experimentally.

The second original contribution of this thesis is to develop two new methods for computing the NNMs of damped systems. The first method solves the partial differential equations (PDEs) governing the geometry of the NNM. The PDEs are recognized as hyperbolic, and it is shown that they require appropriate numerical treatments including specific boundary conditions. The proposed method combines a streamline upwind Petrov-Galerkin finite-element formulation with a resolution strategy based on annular domains to grow sequentially the manifold. The algorithm is demonstrated using a wide variety of systems ranging from two-degree-of-freedom to multi-degree-of-freedom nonlinear systems with linear and nonlinear damping. The applicability of the algorithm to complex real-life structures is demonstrated using a full-scale aircraft. The second method presented in this work computes a NNM as a collection of trajectories defined with boundary value problems (BVPs). The method has the distinctive advantage that it does not rely on a parameterization of the NNM. It is demonstrated on two-degree-of-freedom examples featuring linear and nonlinear damping.